Dynamic Capillary Pressure in Porous Media: Origin of the Viscous-Fingering Length Scale

D. A. Weitz,[(1)] J. P. Stokes,[(1)] R. C. Ball,[(2)] and A. P. Kushnick[(1)]
[(1)] *Exxon Research and Engineering, Annandale, New Jersey 08801*
[(2)] *Cavendish Laboratory, Cambridge, England*
(Received 25 September 1987)

We measure the velocity dependence of the capillary pressure, $\Delta P_c(v)$, between two fluids in a porous medium. At zero velocity, the interface is pinned, and a critical ΔP_c must be achieved before the interface moves. For nonzero velocities, there is an additional dynamic component to ΔP_c, which scales as $v^{1/2}$ when a wetting fluid displaces a nonwetting fluid. We suggest that this dynamic component of ΔP_c can stabilize viscous fingers, and obtain excellent agreement with experiment.

PACS numbers: 47.55.Mh, 47.20.−k, 47.55.Kf

The viscous-fingering instability has been the subject of study for over thirty years[1-3] because of both its fascinating underlying physics and its potential technological importance in many diverse areas involving fluid flow through porous media. The instability arises when a fluid of high viscosity is displaced by a fluid of much lower viscosity. As the displacement velocity increases, the interface between the two fluids becomes unstable to the formation of fingers, resulting in very complex and interesting patterns in the fluid interface.

Our understanding of the viscous-fingering problem has advanced considerably in recent years. In part, this is due to the progress achieved in our understanding of the fingering instability in the Hele-Shaw geometry.[4] The equations describing the fluid flow are similar to those for a porous medium, yet the geometry is much simpler and therefore more amenable to detailed analysis. In part, this is also due to the progress achieved following the observation[5] that these same equations can be mapped onto those describing diffusion-limited aggregation (DLA).

Nevertheless, there remain several fundamental, unresolved questions. Among the most important are those involving the boundary condition at the interface between the two fluids, where the relative ability of the porous medium to be wetted by each of the fluids determines the capillary pressure at the interface. These wetting effects play a critical role in governing the essential physics,[6] despite the fact that fingering occurs at high capillary number, N_{Ca}, when the viscous forces dominate over the capillary forces. In the nonwetting case, when the displaced fluid preferentially wets the porous medium, the individual finger widths are comparable to the pore size, independent of flow velocity or capillary number. In this case, DLA provides an excellent description of the patterns formed,[7] since the finger width can be equated to the walker size. In contrast, in the wetting case, when the displacing fluid preferentially wets the porous medium, the fingering behavior is markedly different. Here, the width of the individual fingers, w, is found to be much larger than any characteristic length scale of the porous medium. Furthermore, scaling behavior is found so that $w \simeq (\kappa/N_{Ca})^{-1/2}$, where κ is the inverse resistance to flow or permeability of the porous medium, and $N_{Ca} = \mu U/\gamma$, where μ is the viscosity of the more viscous fluid, γ the surface tension between the two fluids, and U the average fluid velocity. The origin of this new length scale and its scaling behavior have, in fact, been outstanding puzzles in the behavior of the viscous-fingering instability in porous media for over thirty years.[1,8]

An important key to the resolution of these puzzles lies in the behavior of the capillary pressure. However, to date there have existed no experimental data to guide the theoretical treatment of both the boundary conditions as well as the choice of the continuum equations which describe the fluid motion. In this Letter we present the results of experiments which directly measure the capillary pressure drop across the interface, ΔP_c, and its dependence on the velocity of the interface. We show that it has two important features: At low velocities, there is a pinning pressure which must be overcome before the interface can move; at high velocities, there is a nonlinear dependence on velocity and, in fact, ΔP_c can actually change sign at sufficiently large velocities. This behavior of the capillary pressure has not been included in any analysis of the viscous-fingering instability to date. Here, we suggest that it can fundamentally change the nature of the instability. Using our results we present a new analysis of the instability that accounts for both the scaling and the characteristic length scale of the finger width.

Our experiment is a direct measure of the capillary pressure between two fluids as the interface moves through a porous medium. Our porous medium is formed from a 5-mm-diam tube filled with lightly sintered glass beads of 0.5-mm diameter. We ensure that the glass is preferentially water wetted by heating it in a 0.5*M* solution of nitric acid prior to each experiment. The tube is initially filled with the nonwetting fluid, decane, which is displaced by the wetting fluid, water. The total pressure drop across the porous medium is mea-

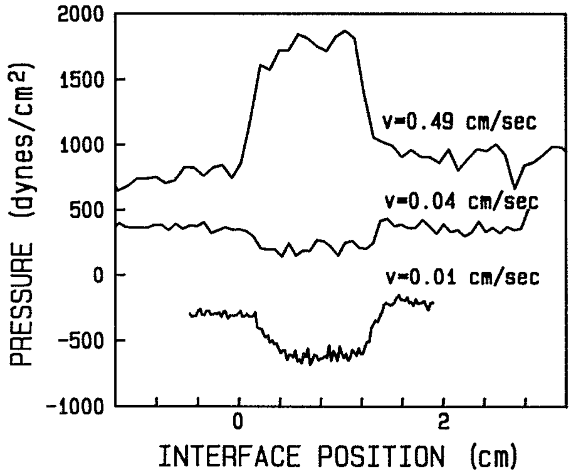

FIG. 1. Pressure across a 1.5-cm-long tube of porous medium as a function of position as nonwetting oil is displaced by wetting water at three different velocities. The jump in the pressure occurs as the interface moves through the porous medium, and reflects the magnitude of the capillary pressure.

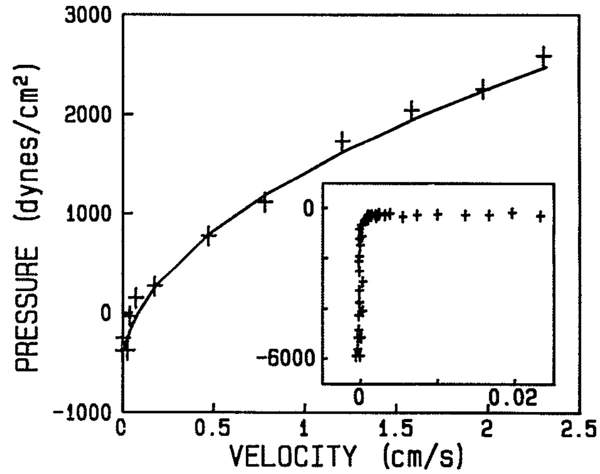

FIG. 2. Capillary pressure as a function of velocity. The solid line is a fit to the data. Inset: The large-pinning regime and the sudden jump in velocity which occurs upon depinning.

sured with a piezoelectric pressure transducer, while the pressure drop across a small capillary tube upstream from the porous medium is also measured to determine the velocity independently. The permeability of this capillary tube is chosen such that the flow is velocity controlled at all but the smallest velocities. The velocity is changed by variation of the pressure head of the displacing fluid. The two fluids used have closely matched viscosities, which ensures that the pressure drop is independent of the position of the interface in the tube and eliminates any viscous instability.

In the absence of an interface in the porous medium, the viscous pressure drop across the tube is $\Delta P_v = \mu v L/\kappa$, where L is the length of the tube, and v is the measured velocity of the flow. An interface results in an additional pressure drop, $\Delta P_c(v)$. To ensure that the viscous pressure drop does not completely dominate the total pressure at higher velocities, the length of the porous medium is limited to 1.5 cm. In Fig. 1 we show the pressure drop across the porous medium as a function of interface position for different velocities. The pressure jumps as the interface passes through the porous medium. The height of this jump reflects the magnitude of the capillary pressure. Not only is ΔP_c dependent on velocity, but at sufficiently high v, it changes sign. We note that ΔP_c is unchanged after the interface has passed through the porous medium, which indicates that κ is unchanged and suggests that virtually all the oil is displaced by the water.

The results are summarized in Fig. 2, where we plot ΔP_c as a function of velocity. At low velocities, $\Delta P_c < 0$, reflecting the fact that the wetting properties cause the water to imbibe into the porous medium. However, the "strength" of this imbibition decreases as the velocity increases, and at sufficiently high velocity, $\Delta P_c > 0$, so that an additional driving pressure is required to force the interface through the porous medium. As additional important feature is illustrated by the low-velocity behavior of ΔP_c, shown in the inset of Fig. 2. There is a relatively large range of pressure over which the interface does not move, or is "pinned." Once this pinning pressure is overcome, there is a sharp jump in the velocity, with very little increase in ΔP_c. One important consequence of the pinning is that the static capillary pressure, $\Delta P_c(0)$, is not well defined.

While the behavior of ΔP_c is quite complex, we can parametrize its velocity dependence. We assume that the magnitude of the capillary pressure is set by the static value, $\Delta P_c(0) \approx \gamma/r_{\mathrm{th}}$, where r_{th} is some characteristic throat radius. While the pinning of the interface makes this somewhat difficult to define precisely, we expect the static value to represent the critical depinning pressure at which the interface just starts to move. We then hypothesize a nonlinear dynamic contribution to the capillary pressure, so that $\Delta P_c(v) \approx \gamma/r_{\mathrm{th}}(-1+KN_{\mathrm{Ca}}^{x})$, where K is a constant and x an exponent to be determined from the data. We use a nonlinear least-squares fit to the data to obtain the results shown by the solid curve in Fig. 2. We obtain $\gamma/r_{\mathrm{th}} \approx -550 \pm 50$ dyn/cm^2, $K \approx 300 \pm 50$, and $x \approx 0.5 \pm 0.1$, where the errors reflect an estimate of our confidence in the values determined from the data. The value obtained for the static capillary pressure is roughly equal to the critical pressure at which the interface is depinned and starts to move. We obtain similar behavior using a tube filled with beads of a different size and confirm that the magnitude of $\Delta P_c(v)$ scales with bead size, and so it is set by γ/r_{th}, while both K and x are independent of bead size.

We can now reexamine the origin of the viscous-fingering instability in light of these measurements of the behavior of the capillary pressure. The traditional view[1] held that the viscous instability competes with the stabilizing force of the surface tension of the interface as it is deformed by the finger. The essential physics of this is captured in a dimensional analysis whereby the length scale is determined by matching of the viscous and capillary pressure drops. This gives the observed scaling of the finger width, $w \simeq (\kappa/N_{\mathrm{Ca}})^{1/2}$, but predicts a finger width much smaller than observed. This has led to the hypothesis of an effective surface tension, substantially larger than γ, that varies with the large-scale curvature of the interface.[1,8] A more recent computer simulation of the Hele-Shaw equations, which included large permeability fluctuations, found a similar scaling for the finger width.[9] However, we are unable to find any experimental evidence whatsoever that the capillary pressure depends on the overall radius of curvature of the interface. Instead, we find that when the interface is stationary, $\Delta P_c(v)$ is not well defined but exhibits a critical depinning pressure determined by γ/r_{th}.

In contrast, $\Delta P_c(v)$ will vary along the interface of a *moving* finger since the local velocity of the interface varies with position. We therefore hypothesize an entirely new stabilization mechanism which incorporates the experimentally observed behavior of $\Delta P_c(v)$. Since the dynamic component of the capillary pressure makes it more difficult to move the interface the greater its velocity, we assume that this provides the stabilizing force. We thus ignore the static component of $\Delta P_c(v)$ entirely as it is constant and independent of position, and can therefore play no role in the determination of any dynamic stability. Again, we use a dimensional analysis to capture the essential physics of this mechanism, and we equate the viscous pressure drop to the *dynamic* component of the capillary pressure drop and solve for the length scale. This gives $w \approx 2\gamma\kappa K N_{\mathrm{Ca},t}^{x-1}/r_{\mathrm{th}}$, where $N_{\mathrm{Ca},t}$ refers to the velocity measured at the tip of the finger, rather than the far-field, average velocity.

We can test this hypothesis by using data obtained with a Hele-Shaw cell filled with glass beads identical to those used in these experiments.[6] Previous analysis of such data showed that w scaled as $N_{\mathrm{Ca}}^{-1/2}$, where the capillary number referred to the average fluid velocity. Since the important parameter in this model is $N_{\mathrm{Ca},t}$, we have reanalyzed this data by examining the video recordings of the fingering and measuring the tip velocity of each finger directly. In Fig. 3 we replot the data but now use $N_{\mathrm{Ca},t}$ of the individual fingers. The measured finger width is normalized by $\kappa^{1/2}$ to include results obtained with three different bead sizes. We again find a similar scaling behavior, $w \simeq (\kappa/N_{\mathrm{Ca},t})^{1/2}$. By comparison, since we measure $x \approx 0.5$, our model predicts $w \simeq N_{\mathrm{Ca},t}^{-1/2}$. Furthermore, since $\kappa \simeq r_{\mathrm{th}}^2$, we also predict that $w \simeq \kappa^{1/2}$. Thus our predicted scaling behavior is in excellent agreement with our experiments.

The magnitude of the length scale in our model is determined by the magnitude of the *dynamic* component of ΔP_c and is reflected by the constant K, which we measure independently in our experiments. To test the agreement with the viscous-fingering results, we rewrite our prediction for the finger width as $w \approx 2(\gamma/r_{\mathrm{th}}) \times (\kappa/\gamma) K N_{\mathrm{Ca},t}^{-1/2}$. We assume that the change from decane ($\mu = 1$ cP) to mineral oil ($\mu = 190$ cP) affects only γ and N_{Ca}, but not the form of $\Delta P_c(v)$. Thus our fit to the data in Fig. 2 gives $\gamma/r_{\mathrm{th}} \approx 550$ dyn/cm^2 and $K \approx 300$, and independent measurements give $\gamma \approx 41$ dyn/cm, and $\kappa \approx 2\times10^{-6}$ cm^2. Then, for $N_{\mathrm{Ca},t} = 8.1\times10^{-4}$, we predict $w \approx 0.56$ cm. By comparison, from Fig. 3, we measure $w \approx 1.3$ cm. We consider the agreement quite reasonable given the approximate nature of our model.

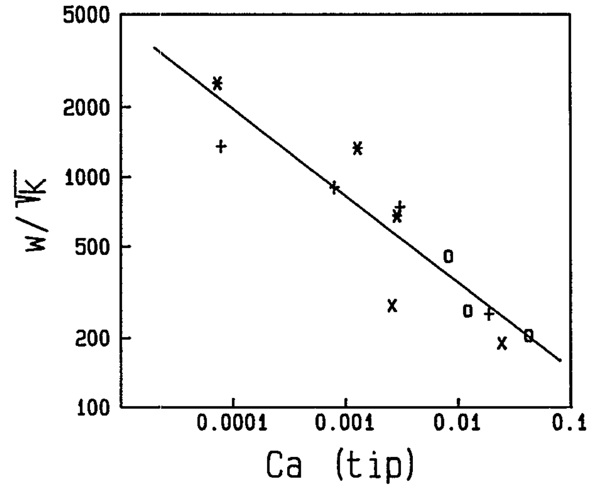

FIG. 3. Scaling behavior of the viscous-finger width in the wetting case, with the width normalized by $\sqrt{\kappa}$, and $N_{\mathrm{Ca},t}$ the capillary number which refers to the velocity of the tip of the individual fingers. The slope of the solid line is -0.5.

These results suggest that it is the dynamic component of the capillary pressure which is crucial in the viscous-fingering instability in the wetting case. We emphasize, however, that despite the strong supporting evidence, these experiments are not a proof of our model. We note, for example, that the static component of the capillary pressure represents an essential singularity in any solution of the continuum equations which describe the Hele-Shaw cell, and can thus not be ignored. However, it is not clear which continuum equations are appropriate for a porous medium, given the behavior of $\Delta P_c(v)$ observed, with its pinning at low velocity and its dynamic dependence at higher velocities. It is possible that a microscopic treatment of the fluid flow in the complex, disordered geometry will lead to an alternative explanation of the new length scale. However, in the absence of such a treatment, we feel that the model proposed here, which is based on a continuum treatment of the flow, represents a reasonable account of the physics and is well supported by the experimental data.

By way of conclusion, we note that in this paper we have presented experimental data on the behavior of ΔP_c and have discussed its consequences on the viscous-fingering instability. We have not considered at all the very interesting and important question of the physical origin of the observed behavior of $\Delta P_c(v)$. The low-velocity pinning behavior is clearly due to the strong disorder of the porous medium. In fact, one can map the movement of an interface at these velocities onto the random-field problem,[10,11] analogous to other nonlinear transport problems such as, for example, sliding charge-density waves (CDW). Thus the interface velocity corresponds to the CDW current while ΔP_c corresponds to the voltage. Of course, the center of the pinned region of ΔP_c is offset from zero by comparison to the CDW voltage because of the imbibition of the wetting fluid at very low velocities.

The dynamic effects observed at high velocities bear some similarity to those seen for the capillary pressure drop in a smooth tube with a moving bubble of a nonwetting fluid, first discussed by Bretherton.[12] However, to our knowledge, there has been no analytic treatment of the moving contact line for flow in a smooth tube. Experimentally, the contact angle on a plane surface is known to exhibit both hysteresis and a dependence on velocity.[13] Furthermore, in a capillary tube, Hoffman[14] has studied the interface of a wetting fluid advancing into air and finds that the cosine of the contact angle, and thus ΔP_c, change sign as the velocity is increased. However, in a capillary tube, ΔP_c changes sign at a value of N_{Ca} 2 orders of magnitude larger than in the porous medium. This suggests that the strong disorder in the porous medium also plays a major role in causing the velocity dependence of ΔP_c. Clearly more work is required to account fully for the origin of the observed behavior, and further experiments are under way in our laboratory to address these important and fascinating questions.

We acknowledge useful and informative discussions with Mac Lindsay, Marko Robbins, and Eric Herbolzheimer.

[1]R. L. Chouke, P. van Meurs, and C. van der Poel, Trans. AIME, **216**, 188 (1959).

[2]P. G. Saffman and G. I. Taylor, Proc. Roy. Soc. London A **245**, 312 (1958).

[3]G. M. Homsy, Annu. Rev. Fluid Mech. **19**, 271 (1987).

[4]D. Bensimon, L. P. Kadanoff, S. Liang, B. I. Shraiman, and C. Tang, Rev. Mod. Phys. **58**, 977 (1986) and references therein.

[5]L. Paterson, Phys. Rev. Lett. **52**, 1621 (1984).

[6]J. P. Stokes, D. A. Weitz, J. P. Gollub, A. Dougherty, M. O. Robbins, P. M. Chaikin, and H. M. Lindsay, Phys. Rev. Lett. **57**, 1718 (1986).

[7]K. Maloy, J. Feder, and T. Jossang, Phys. Rev. Lett. **55**, 2688 (1985).

[8]E. Peters and D. Flock, Soc. Pet. Eng. J. **21**, 249 (1981).

[9]G. Li and L. M. Sander, Phys. Rev. A **36**, 4551 (1987).

[10]J. P. Stokes, A. Kushnick, and M. O. Robbins, to be published.

[11]J. Koplick and H. Levine, Phys. Rev. B **32**, 280 (1985).

[12]F. P. Bretherton, J. Fluid Mech. **10**, 166 (1961).

[13]E. B. Dussan V., Annu. Rev. Fluid Mech. **11**, 371 (1979).

[14]R. L. Hoffman, J. Colloid Interface Sci. **50**, 228 (1975).