Triple Junction at the Triple Point Resolved on the Individual Particle Level

M. Chaudhuri,1,2,3 E. Allahyarov,3,4 H. Löwen,5 S. U. Egelhaaf,5,* and D. A. Weitz1,2
1Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
2School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA
3Institute for Theoretical Physics II: Soft Matter, Heinrich Heine University, 40225 Düsseldorf, Germany
4Theoretical Department, Joint Institute for High Temperatures, Russian Academy of Sciences (IVTAN), Moscow 125412, Russia
5Condensed Matter Physics Laboratory, Heinrich Heine University, 40225 Düsseldorf, Germany
(Received 11 May 2017; published 21 September 2017)

At the triple point of a repulsive screened Coulomb system, a fcc crystal, a bcc crystal, and a fluid phase coexist. At their intersection, these three phases form a liquid groove, the triple junction. Using confocal microscopy, we resolve the triple junction on a single-particle level in a model system of charged PMMA colloids in a nonpolar solvent. The groove is found to be extremely deep and the incommensurate solid-solid interface to be very broad. Thermal fluctuations hence appear to dominate the solid-solid interface. This indicates a very low interfacial energy. The fcc-bcc interfacial energy is quantitatively determined based on Young’s equation and, indeed, it is only about 1.3 times higher than the fcc-fluid interfacial energy close to the triple point.

According to the traditional Gibbs phase rule of thermodynamics [1], in a one-component system up to three phases can coexist. Their coexistence is represented by a triple point in the temperature-pressure phase diagram and a triple line in the temperature-density phase diagram. At triple conditions, the three phases are in mutual mechanical, thermal, and chemical equilibrium. The three possible interfaces only occur at the same time if the interfacial energies are similar; if an interfacial energy is larger than the sum of the other two, this interface is unstable and the third phase intervenes. When the three interfaces intersect, they form an interfacial line, the triple junction line (which is a point in the slice shown in Fig. 1). Triple junctions have been studied on the macroscopic level, for example, in metals where liquid lenses form on top of a crystallite surrounded by coexisting vapor [2,3]. Another classical example is the triple point of water where vapor, liquid water, and ice coexist. Such a gas-liquid-solid triple point involves two disordered and one ordered phase and can exist in systems governed by sufficiently long-ranged attractive interparticle interactions [4–6]. In contrast, the coexistence of a fluid and two different solids involves two ordered structures and hence an interface between two crystallites that are not commensurate. The corresponding phase behavior has been studied in suspensions of charged colloids [7–16], star polymers [17], microgels [18], and dusty plasmas [19–27]. In these systems, a fcc crystal, a bcc crystal, and a fluid can coexist. Colloids have long been used as model systems to explore such situations, including nucleation [24–27], crystallization [28–32], melting [33–38], defects [39], glass transition [44–48], solid-liquid interfaces [40–44], solid-solid phase transformations [45–49], as well as the microscopic response to external forces [49–52].

Using charged colloids [53,54], we investigate the triple junction at the triple point on an individual-particle level, i.e., including the smallest relevant length scale. At the triple point, we find a deep and tight fluid groove between the two solid phases and a very broad solid-solid interface. This indicates a small solid-solid interfacial energy and hence a considerable effect of thermal fluctuations. Indeed, a quantitative determination of the interfacial energy using Young’s equation confirms this suggestion.

The interactions of highly charged colloids in the presence of small ions can be described by a purely repulsive screened Coulomb (or Yukawa) effective pair interaction $U(r) = (Q^2/4\pi\epsilon_0\epsilon r) \exp(-r/\lambda)$, with the particle charge $Q = Ze$, the elementary charge $e$, the permittivity of the vacuum $\epsilon_0$, the relative permittivity $\epsilon$, and the Debye screening length $\lambda$ [11–13]. Phase space is completely parametrized by the Coulomb coupling parameter $\Gamma = Q^2/(4\pi\epsilon_0\epsilon a k_B T) = Z^2\lambda_B/a$ and the screening

FIG. 1. Schematic representation of a triple junction involving two different solids, a fcc (green) and a bcc (red) phase, as well as a fluid (blue). A fluid groove can exist with a triple junction at its tip. The dihedral angle at the tip $\beta = \beta_{\text{fcc}} + \beta_{\text{bcc}}$ where $\beta_{\text{fcc}}$ and $\beta_{\text{bcc}}$ are indicated. The depths of the groove, $h_{\text{fcc}}$ and $h_{\text{bcc}}$, are also indicated.
parameter \( \kappa = a/\lambda \), where \( k_B T \) is the thermal energy, \( \lambda_B = e^2/(4\pi\epsilon_0 k_B T) \) the Bjerrum length, \( a = \rho^{-1/3} \) the mean interparticle distance, and \( \rho \) the particle number density. For this system, using molecular dynamics simulations the triple point was located at \( \Gamma \approx 3500 \) and \( \kappa = 6.90 \) [13].

We use charged fluorescently labeled PMMA spheres coated with polyhydroxystearic acid with a radius \( R \approx 0.80 \mu m \), as determined by dynamic light scattering. They are suspended in a nonpolar solvent mixture of decalin \( (\epsilon = 2.1, \ \rho = 0.881 \ g/ml, \ \text{refractive index} \ n = 1.48) \) and tetrachloroethylene \( (\epsilon = 2.5, \ \rho = 1.623 \ g/ml, \ n = 1.51) \), with a ratio of 6:5 (by volume). This mixture has a viscosity \( \eta \approx 1.29 \ \text{mPa s} \) as determined by rheology, a relative permittivity \( \epsilon \approx 2.3 \), a density that closely matches the particle density, and a refractive index very similar to the one of the particles.

In nonpolar solvents charges do not readily dissociate. However, they can be stabilized in the cores of reverse surfactant micelles [55]. They favor the dissociation of charges from the particle surface resulting in negatively charged particles and charge screening by the charged reverse micelles, similar to the mechanism in polar solvents. Furthermore, two neutral micelles can reversibly interact to yield two oppositely charged micelles; roughly 1 in \( 10^5 \) micelles acquires a single elementary charge in this way [55].

We use 20 mM diolyl sodium sulfosuccinate (AOT), which is well above the estimated critical micelle concentration of about 1 mM [55–57], and yields reverse micelles with an essentially concentration-independent radius \( R_m \approx 1.5 \) nm [55,58]. At this AOT concentration the conductivi\( \approx 80 \) pS/cm, as measured using an immersion probe, and thus the number density of charged micelles, and hence ions, is estimated to be \( \rho_m = 6\pi\eta R_m c/e^2 \approx 10^{19} \) m\(^{-3} \) (and the number density of all, that is charged and uncharged, micelles is higher by a factor of about \( 10^3 \)). This results in an estimate of the screening length \( \lambda = 1/\sqrt{4\pi\lambda_B \rho_m} \approx 0.5 \) \( \mu m \), with \( \lambda_B \approx 24 \) nm. Electrophoretic light scattering measurements yield the normalized zeta potential \( \zeta / k_B T \approx 3.6 \). The particle charge number can be estimated within the Derjaguin-Landau-Verwey-Overbeek theory, \( |Z| = |e^{R/\lambda}/(1 + R/\lambda)|\zeta / k_B T \approx 900 \) [59]. Because of charge saturation, this is only a crude upper estimate [60].

We use a sample at the triple point to determine the particle charge with high accuracy. A concentrated sample is prepared and less dense samples obtained by adding supernatant. The sample with three coexisting phases is identified using confocal microscopy [61]. The three coexisting phases have slightly different particle number densities, \( \rho \approx 0.030 \) \( \mu m^{-3} \) (fluid), 0.035 \( \mu m^{-3} \) (bcc), and 0.040 \( \mu m^{-3} \) (fcc) (Fig. 2, filled stars), which is consistent with the small size of the coexistence region predicted for highly charged particles [11]. At the triple point, \( \Gamma \approx 3500 \) and \( \kappa = 6.90 \) [13]. This suggests a screening length \( \lambda \approx 0.45 \mu m \) consistent with the estimated \( \lambda \approx 0.5 \mu m \) and a particle charge \( |Q| \approx 670e \), which is below the crude upper estimate \( |Q| \approx 900e \). Thus, the values determined from the sample at the triple point are consistent with those based on the sample composition. It also confirms that the particles are highly charged. This can be quantified by the reduced contact value of the pair potential consisting of repulsive Yukawa and hard core interactions, \( \Gamma^* = Z^2(\lambda B/2R)(1 + R/\lambda)^{-2} \approx 900 \) [8,11]. Thus, \( \Gamma^* \gg 20 \) and hard core interactions are negligible [11]. The particles hence can be treated as highly charged, pointlike particles with purely repulsive screened Coulomb interactions.

Having determined the particle charge \( Q \approx -670e \) and the screening length \( \lambda \approx 0.45 \mu m \), using these parameters we can locate the samples in the phase diagram (Fig. 2). The observed phase behavior is consistent with previous experimental findings and theoretical predictions for a repulsive screened Coulomb system [9–15,23,62–67].

The sample with three coexisting phases is investigated in more detail in the following. Confocal microscopy images taken 20 \( \mu m \) from the cover slip show the coexistence of two crystalline solids and a fluid (Fig. 3) and thus fluid-solid interfaces as well as a solid-solid interface. At the solid-solid-fluid triple line a fluid groove starts that is many crystal layers deep. It has a small dihedral angle that appears slightly asymmetric, reflecting the two different crystalline solids. This is different in grain-boundary grooves that are formed between crystallites of the same structure, which have experimentally been observed in hard sphere systems [48,68].

Images of the groove are quantitatively analyzed to retrieve a profile of the groove and to determine the crystal

![FIG. 2. Phase diagram of the purely repulsive Yukawa system as a function of the Coulomb coupling parameter \( \Gamma \) and the screening parameter \( \kappa \). The experimentally investigated samples in the different single-phase regions are represented by + (fcc), ○ (bcc), and × (fluid), and the three coexisting phases in the sample under triple conditions by green stars (fcc), red stars (bcc), and blue stars (fluid). The theoretically predicted fluid-solid coexistence line is indicated by red triangles [13], blue solid circles [14], and green squares [15], and the corresponding analytical expression \( \Gamma = 106e^2/(1 + \kappa + 0.5\kappa^2) \) [23] by a solid line. The theoretically predicted fcc-bcc coexistence is indicated by blue triangles connected by a line. The inset shows the data close to the triple point.](image-url)
structures. This requires us to determine whether a particle belongs to the fluid or one of the solids. The particle locations are extracted from the image stacks by standard algorithms [69], and for each particle the local bond order parameters $\bar{q}_4$ and $\bar{q}_6$ [32,70] are calculated (Fig. 4). The distribution of $\bar{q}_6$ and $\bar{q}_4$ values indicates three populations which can also be identified in the histogram of $\bar{q}_6$ (Fig. 4, right). The $\bar{q}_6$ values hence can be used to guide the classification of the particles [70]. Particles with $\bar{q}_6 \leq 0.19$ are likely to belong to the fluid, particles with $0.19 < \bar{q}_6 < 0.28$ to the bcc crystal, and particles with $\bar{q}_6 \geq 0.28$ to the fcc crystal. A similar assignment is obtained based on the number of neighbors instead of the local bond order parameter $\bar{q}_6$. If the particles are labeled accordingly, the groove and interface separating the fcc and bcc crystals are clearly visible (Fig. 5, bottom). Furthermore, the orientations of the fcc and bcc bulk crystals can be determined; in both cases the (111) plane is oriented horizontally. This analysis also confirms the presence of the triple junction at the intersection of the fcc-fluid, bcc-fluid, and fcc-bcc interfaces (Figs. 3 and 5). While the three phases can be distinguished on a mesoscopic level, this is not the case on a microscopic level; individual particles are observed to spread into the neighboring regions (Fig. 5, top). Fluid particles significantly penetrate into the two crystalline regions, in particular, into the interfaces and the bcc crystal. Moreover, individual particles with bcc structure are found in the fcc and fluid regions. In contrast, the fcc particles are essentially confined to the fcc crystal with very few particles with an fcc-like structure inside the bcc crystal. This distribution of particles is attributed to defects and fluctuations and partially could be due to the ambiguity in the link between crystal structure and $\bar{q}_6$ value, but also the difficulty to assign a crystal symmetry to an individual particle at finite temperature. Furthermore, the fcc-fluid interface shows a high degree of bcc-like ordering, as predicted by simulations [71].

To determine the solid-solid, i.e., fcc-bcc, interfacial energy $\gamma_{fcc-bcc}$, the groove is quantitatively analyzed. The rendered three-dimensional stack is divided into 25 quasi-two-dimensional $x$-$y$ planes with a thickness of about 2.2 $\mu$m [Fig. 6(a)]. These planes are essentially parallel to the crystal planes and represent one particle layer (Fig. 5, bottom). They show grooves with slightly different depths [Fig. 6(b)].
The tip of a groove, i.e., the triple junction, is defined as the particle that is most distant from the bulk fluid but still connected to the bulk fluid through other fluid particles. The structural parameters of the grooves [Fig. 1(b)] are quantitatively determined for each plane [Figs. 6(c) and 6(d)]. The grooves are slightly more than 100 μm wide and about 80 μm deep, which corresponds to about 50 particle diameters or 35 interlayer spacings. Towards the fcc crystal, the depth of the groove, on average \( h_{\text{fcc}} \approx 84 \mu m \), is slightly larger than towards the bcc crystal, on average \( h_{\text{bcc}} \approx 72 \mu m \). We find similarly for the angles; \( \beta_{\text{bcc}} \approx 8.2^\circ \) is slightly larger than \( \beta_{\text{fcc}} \approx 4.9^\circ \), but both are very small. This results in a small dihedral angle \( \beta = \beta_{\text{bcc}} + \beta_{\text{fcc}} \approx 13^\circ \). This is in contrast to findings for grain boundaries between crystallites with the same symmetry where the dihedral angles are larger, about 100° [48].

Mechanical stability of the triple junction line requires Young’s condition to hold [68,72–74]. It links the interfacial free energies \( \gamma \) to the interface intersection angles \( \beta \) (Fig. 1):

\[
\gamma_{\text{fcc-bcc}} = \gamma_{\text{fcc}} \cos(\beta_{\text{fcc}}) + \gamma_{\text{bcc}} \cos(\beta_{\text{bcc}}). \tag{1}
\]

where the subscripts fcc, bcc, and fcc-bcc refer to the fcc-fluid, bcc-fluid, and fcc-bcc interfaces with \( \gamma_{\text{bcc}} \approx 0.12 \frac{k_B T}{a^2} \) and \( \gamma_{\text{fcc}} \approx 0.40 \frac{k_B T}{a^2} \) [44] with an extended comparison of experimental and theoretical values provided by Ref. [43]. The angles refer to the directions of the interfaces close to the triple junction. This equation is based on the assumption that the interfacial energy is isotropic. It was shown to adequately describe the fluid-bcc interfaces in Yukawa systems close to the triple conditions, where the interfacial energy is almost isotropic [44]. In addition, the fluid-fcc interface energy in hard sphere systems has been found to show only a modest dependence on the orientation [75]. Since the fcc-bcc interface has a fluidlike structure with many defects, it appears reasonable to assume that its interfacial energy is not very anisotropic.

Using Young’s equation [Eq. (1)], the fcc-bcc, i.e., solid-solid, interfacial energy \( \gamma_{\text{fcc-bcc}} \) can be calculated. We find \( \gamma_{\text{fcc-bcc}} \approx 0.52 \frac{k_B T}{a^2} = 1.3 \gamma_{\text{fcc}} \) [Fig. 6(e)]. Thus, the interfacial energy is very small, although it involves two solid phases. The small interfacial energy is consistent with the pronounced fluctuations observed at the fcc-bcc interface (Fig. 5): the interface is several layers wide and hence astonishingly broad. The value of the interfacial energy is considered to be reliable as it shows only a very weak sensitivity to uncertainties in the angles \( \beta_{\text{fcc}} \) and \( \beta_{\text{bcc}} \) due to the small size of the angles and the dependence of \( \gamma_{\text{fcc-bcc}} \) on the cosine of the angles, which is close to 1. Thus, the uncertainty in \( \gamma_{\text{fcc-bcc}} \) is mainly related to the definition of the tip of the groove and less to the uncertainties in the determination of the angles \( \beta_{\text{fcc}} \) and \( \beta_{\text{bcc}} \). Nevertheless, also the definition of the tip is not crucial, as argued in the following.

The observed coexistence of the three phases (Fig. 3) implies that none of the three interfacial energies dominates. For example, the sum of the two interfacial energies, \( \gamma_{\text{fcc}} \) and \( \gamma_{\text{bcc}} \), represents the maximum value of \( \gamma_{\text{fcc-bcc}} \) that is still compatible with the existence of a fcc-bcc interface. For larger values, the fcc-bcc interface would be unstable towards an intervening fluid phase, i.e., complete wetting. Therefore, \( \gamma_{\text{fcc}} + \gamma_{\text{bcc}} \) and, following the corresponding argument, \( \gamma_{\text{fcc}} < \gamma_{\text{fcc-bcc}} + \gamma_{\text{bcc}} \). This implies \( \gamma_{\text{fcc}} - \gamma_{\text{bcc}} < \gamma_{\text{fcc-bcc}} < \gamma_{\text{fcc}} + \gamma_{\text{bcc}} \) and hence \( 0.7 \approx \gamma_{\text{fcc-bcc}} / \gamma_{\text{bcc}} \lesssim 1.3 \). This range is so narrow because the bcc-fluid interfacial energy is small, \( \gamma_{\text{bcc}} \approx 0.3 \gamma_{\text{fcc}} \) [44], which has been attributed to the fact that the bcc structure is relatively close to the fluid structure [44]. This narrow range of values for \( \gamma_{\text{fcc-bcc}} \) hence can be established only based on the observation of the coexistence of the three phases. Moreover, the observation of a tight groove (Fig. 3) implies small angles \( \beta_{\text{bcc}} \) and \( \beta_{\text{fcc}} \) whose cosines are about 1. Thus, \( \gamma_{\text{fcc-bcc}} \) is almost the sum of the two solid-fluid interfacial energies, \( \gamma_{\text{fcc-bcc}} \approx \gamma_{\text{fcc}} + \gamma_{\text{bcc}} \approx 1.3 \gamma_{\text{fcc}} \) [Eq. (1)]. The value of \( \gamma_{\text{fcc-bcc}} \) hence is expected towards the upper limit of the above range of values. This semiquantitative argument is based on the observation of individual particles. However, it does not require us to define the tip of the groove; the observation of a tight groove is sufficient. Thus, there is qualitative support as well as semiquantitative and quantitative evidence for a very small fcc-bcc interfacial energy.

To conclude, we investigated suspensions of charged colloids under triple conditions, where a fcc crystal, a bcc crystal, and a fluid coexist. The fcc-bcc interfacial energy was found to be about 1.3 times higher than the fcc-fluid interfacial energy close to the triple point with \( \gamma_{\text{fcc-bcc}} \approx 0.52 \frac{k_B T}{a^2} \). Thus, the fcc-bcc interfacial energy...
is very small, despite the fact that two solid phases are involved. This is consistent with the observation of broad interfaces and indicates the importance of thermal fluctuations also for solid-solid interfaces. Our quantitative findings and qualitative arguments suggest that a small solid-solid interfacial energy not only occurs in systems with charged particles but also in other systems with soft interactions exhibiting a triple point. Furthermore, also in atomic or molecular systems, e.g., metals [76–79], similar values might be found if expressed in dimensionless units. Our finding hence might in general apply to triple points involving a fluid and two solids. They might also be extended to more complex conditions, such as the presence of shear or other external fields [80–82].

We thank F. Spaepen, A. G. Yodh, and T. Palberg for very useful discussions and P. Maßhoff for help with the figures. M. C. is supported by a Marie-Curie international outgoing fellowship within the European Union’s 7th Framework Program (Grant No. 327168). We acknowledge support by the Deutsche Forschungsgemeinschaft (Grants No. AL2058/1-1, No. LO418/19-1, and No. EG268/6-1). Part of this work was supported by the Harvard MRSEC (DMR-1420570) and NSF (DMR-1310266), and used the experimental facilities at Centre for Nanoscale Systems (CNS), a member of the National Nanotechnology Infrastructure Network (NNIN), which is supported by the National Science Foundation (ECS-0335765).

*Corresponding author.

stefan.egelhaaf@uni-duesseldorf.de


The samples are filled in glass capillaries with a size of about 15×5×0.2 mm³ and left for 2 days to allow for equilibration before volumes of 145×145×60 μm³, corresponding to 512×512×478 voxels, and located 20 μm from the capillary wall are imaged using a confocal microscope (Leica TCS SP5 with a 40× oil-immersion objective).


[J. C. Crocker and D. G. Grier, Methods of digital video microscopy for colloidal studies, J. Colloid Interface Sci. 179, 298 (1996).]

[P. R. ten Wolde, M. J. Ruiz-Montero, and D. Frenkel, Simulation of homogeneous crystal nucleation close to coexistence, Faraday Discuss. 104, 93 (1996).]


[S.-C. Lin, M.-W. Liu, M. P. Gururajan, and K.-A. Wu, Modified Young’s equation for equilibrium dihedral angles of grain boundary grooves in thin films at the nanoscale, Acta Mater. 102, 364 (2016).]


