Actin Filament Length Tunes Elasticity of Flexibly Cross-Linked Actin Networks


ABSTRACT Networks of the cytoskeletal biopolymer actin cross-linked by the compliant protein filamin form soft gels that stiffen dramatically under shear stress. We demonstrate that the elasticity of these networks shows a strong dependence on the mean length of the actin polymers, unlike networks with small, rigid cross-links. This behavior is in agreement with a model of rigid filaments connected by multiple flexible linkers.

INTRODUCTION

The actin cytoskeleton is a composite intracellular biopolymer network. To tune the mechanical properties of the cytoskeleton for such diverse processes as cell division, locomotion, and shape, a large number of actin binding proteins organize network structure (1). Nucleating and capping proteins regulate the polymerization of monomeric actin into filamentous actin (F-actin). Cross-linking proteins bind the actin filaments together to form elastic gels or bundle structures, such as in stress fibers and filopodia. Motor protein assemblies control tension within the networks by pulling on actin filaments cross-linked to the network (2–4). Even though the important molecular components are known, relatively little is understood of how this large ensemble of proteins collectively contributes to the mechanical response of the cytoskeleton.

The complex and composite structure of the cytoskeleton makes studying the origins of the mechanical response difficult. One approach has been to study reconstituted in vitro F-actin networks in the presence of purified binding proteins (3,5–9). Reconstituting the network allows precise control of its chemical composition and systematic investigation of its properties. A ubiquitous feature of these networks is that they stiffen with increasing applied stress. When F-actin is cross-linked by small rigid cross-links, the stiffening arises from the properties of the filaments themselves. F-actin is a semiflexible polymer; the persistence length is 17 μm (10). Thermal fluctuations cause transverse bending in the F-actin, which decreases its end-to-end distance. Application of a force stretches out these fluctuations. For small extensions, the force is proportional to the extension, whereas for large extensions approaching the contour length, the force diverges, leading to strain-stiffening (11).

Both the linear and nonlinear network elasticities are consistent with the theoretical predictions for a network of semiflexible polymers, provided the deformation is affine (5,7,12). However, this picture of network elasticity implicitly assumes that the elasticity is controlled by one component, the actin filaments. It ignores any contribution of the cross-linking proteins; these can be both large and compliant, and therefore can themselves contribute to the elasticity.

One example of a large and flexible cross-link is filamin, which is abundant in cells. Filamin cross-links F-actin into orthogonal networks in the cortex, connects F-actin to integrins, and may play a role in mechanotransduction (13–17). F-actin networks cross-linked by filamin exhibit a mechanical response that is qualitatively different from networks formed with rigid cross-links (6,8,18,19). Filamin-F-actin networks are compliant, weakly elastic solids. Nevertheless, they can support large shear stresses because of their pronounced nonlinear strain-stiffening. Their nonlinear behavior is inconsistent with predictions for an affinely deformed network with rigid cross-links (5,6,19). In comparison to networks with rigid cross-links, networks cross-linked by filamin exhibit mechanical properties that more closely mimic the properties of cells (3,6,18).

Recent experimental reports show that the unusual nonlinear elasticity of these networks is consistent with a model of rigid polymers connected by multiple flexible cross-links (19–21). The rigid polymer of length L constrains the deformation profile of the n flexible cross-links bound along its length, as shown in Fig. 1; thus, polymer length is predicted to be an important parameter controlling the linear and nonlinear properties of the network (21). Indeed, the linear viscoelasticity and rupture stress of F-actin networks cross-linked by filamin are sensitive to the addition of gelsolin (19,22), an actin-capping and severing protein that shortens the average filament length. While
otherwise noted, and control network microstructure by varying the molar
concentration of filamin-gelsolin-F-actin networks.

We form networks with an actin concentration,
and we show that the linear and nonlinear elastic behavior of
F-actin is indeed tuned by varying
the ratio of filamin dimers/actin monomers, \( R_f \).
We regulate the actin-filament length distribution with gelsolin. The molar ratio of gelsolin/actin monomers,
\( R_{GC} \), sets the mean actin filament length (26).
Samples are prepared by mixing solutions of \( 10 \times \) polymerization buffer (20 mM Tris-HCl, 20 mM MgCl₂, 1 M KCl, 2 mM DTT, 2 mM CaCl₂, and 5 mM ATP, pH 7.5), gelsolin, filamin, and G-actin.

For rigidly cross-linked networks, biotinylated actin monomers are incorporated in actin filaments at a molar ratio of biotinylated G-actin to nonbiotinylated G-actin, \( R_b \). Cross-linking is mediated by NeutrAvidin protein.
Samples are prepared by mixing \( 10 \times \) polymerization buffer to gelsolin, biotinylated G-actin, and G-actin. After 3 min, NeutrAvidin at a 1:1 molar ratio to biotinylated actin is gently mixed in.

The sample is loaded into a microscopy chamber, consisting of two
coverslips with a 1-mm spacer, or between rheometer plates and polymerized for 1 h at 25 °C.

**Characterization of F-actin length distribution**

To characterize the actin-filament length distribution, we polymerize
0.3 mg/mL F-actin in the presence of gelsolin. After 1 h, the filaments are labeled and stabilized with a 1:1 molar ratio of Alexa-488 phalloidin and incubated at 25 °C for 30 min. The filaments are diluted to a concentration of 2 nM, and 5 μL of the suspension is pipetted onto a coverslip functionalized with poly(acrylamide-co-diethylaminoethylamide mononitrom chloride).
A second coverslip is placed on top and the sample sealed. Nearly all filaments stick to the coated coverslip. Immobilized filaments are imaged using a confocal microscope (model No. TCS SP5; Leica, Wetzlar, Germany; image pixel size is 160 nm. Filament contour lengths, \( L \), are measured manually with ImageJ (National Institutes of Health, Bethesda, MD); the minimum distance measurable using this method is 0.5 μm. For each gelsolin concentration, the width of the distribution of filament lengths

\[
\sigma = \sqrt{\langle L^2 \rangle - \langle L \rangle^2}
\]

is nearly equivalent to the mean (Fig. S1 in the Supporting Material).
For network formation, we polymerize F-actin in the presence of gelsolin and filamin. In previous work, addition of α-actinin to gelsolin-regulated F-actin narrowed the width of the length distribution without significantly affecting the mean length \( L \) (27). Similarly, we expect that filamin should not significantly change the values of \( L \) we measure here.

**Imaging**

For confocal microscopy, samples are fluorescently labeled by polymerizing in the presence of 0.6 μM Alexa-488 phalloidin and examined (model No. TCS SP5; Leica). For transmission electron microscopy, a 10 μL drop of assembled network is applied to a 400-mesh carbon-coated nickel grid and incubated for 30 s, stained with 1% uranyl acetate, rinsed by passing a drop of distilled water over the grid, then air-dried and imaged (model No. 2100; JEOL USA, Peabody, MA).

**Rheology**

We use a stress-controlled rheometer with 40-mm stainless steel parallel plates and a 160-μm gap (AR-G2; TA Instruments North America, New Castle, DE; or C-VOR; Bohlin Instruments/Malvern Instruments, Malvern, Worcestershire, UK). We polymerize samples in situ and use a solvent trap, applying a thin layer of low-viscosity mineral oil around the sample to minimize evaporation. We confirm the results are independent of gap and reproducible within and between different protein preparations.

The linear viscoelastic response is measured by applying a frequency-dependent, sinusoidal stress, \( \sigma \sin(\omega t) \), and measuring the strain,
Upon adding a small amount of gelsolin, the unregulated F-actin has a mean length of \( L \). This is consistent with a model where each gelsolin molecule associates with one actin filament. Each actin monomer adds 2.7 nm to the filament length \( (30) \), so that 1 \( \mu \)m of filament is composed of 370 monomers, predicting \( L = (370 R_G)^{-1} \). Some inactivation of gelsolin during storage may account for the slightly larger observed filament lengths compared to the prediction. These findings are consistent with previous studies of actin-filament length distributions \( (26,27,31) \).

\[
\gamma_s \sin(\omega t + \delta). \quad \text{We maintain } \gamma_s < 2\% \text{ to ensure linear response. The elastic modulus is}
\]

\[
G'(\omega) = \frac{\sigma_0}{\gamma_s} \cos(\delta);
\]

the viscous modulus is

\[
G''(\omega) = \frac{\sigma_0}{\gamma_s} \sin(\delta).
\]

We measure the response in the nonlinear regime with a differential or “prestress” measurement; a small amplitude oscillatory stress, \( \sigma_t \), is superposed on a steady prestress, \( \sigma_s \), to measure the differential modulus,

\[
K^e(\sigma_0, \omega) = \delta \sigma / \delta \gamma|_{\sigma_s}.
\]

The elastic and viscous components are \( K^e \) and \( K^v \), respectively. We confirm there is no time dependence in \( K^e \) at various levels of prestress and minimal hysteresis in \( K^e(\sigma_s) \) (Fig. S5). In a complementary strain ramp approach, we increase the strain at a fixed rate and measure the resulting stress. Both \( \sigma(t) \) and \( \gamma(t) \) are smoothed using a cubic spline algorithm to compute the differential modulus

\[
K = \frac{d\sigma}{d\gamma}
\]

by applying a numerical derivative to the stress-strain curve.

RESULTS AND DISCUSSION

F-actin length distribution in the presence of gelsolin

Within the cell, the contour lengths, \( L \), of actin filaments are highly regulated. Typical lengths range from 100 nm to a few microns \( (28,29) \). In vitro, high enough concentrations of pure monomeric actin will polymerize spontaneously in the presence of divalent salt and ATP. Yet, these in vitro filaments are typically much longer than those in the cell, with contour lengths that can be up to 50 \( \mu \)m (Fig. S1).

To better mimic the conditions in cells, we use the F-actin capping and severing protein gelsolin to vary the mean length, \( L = \langle L \rangle \), of our in vitro actin filaments. To characterize the filament length distribution in the presence of gelsolin, we image a diluted sample of F-actin stabilized with fluorescent phalloidin (Fig. S1). For the ratio of gelsolin/actin, \( R_G = 0 \), the unregulated F-actin has a mean length of \( L = 14.8 \) \( \mu \)m. Upon adding a small amount of gelsolin, \( R_G = 1.3700 \), the length distribution is dominated by the presence of the gelsolin and \( L \) decreases to 10.4 \( \mu \)m. Increasing \( R_G \) decreases \( L \) further. We find that \( L \) scales linearly with \( R_G^{-1} \), as shown in Fig. 2; it varies as \( L = (330 R_G)^{-1} \), with \( L \) measured in microns. This is consistent with a model where each gelsolin molecule associates with one actin filament. Each actin monomer adds 2.7 nm to the filament length \( (30) \), so that 1 \( \mu \)m of filament is composed of 370 monomers, predicting \( L = (370 R_G)^{-1} \). Some inactivation of gelsolin during storage may account for the slightly larger observed filament lengths compared to the prediction. These findings are consistent with previous studies of actin-filament length distributions \( (26,27,31) \).

Microstructure of filamin-gelsolin-F-actin networks

We form in vitro networks of actin filaments whose lengths are regulated with gelsolin and which are cross-linked by filamin. In vitro, filamin efficiently cross-links F-actin into orthogonal networks, which are soft but support large stresses \( (6,13) \). These networks mimic several key features of cell mechanical properties \( (3,6,18) \). The microstructure of these networks varies as we change \( L \) and the molar ratio of filamin/actin, \( R_F \).

For \( R_F \leq 0.01 \), the networks are a homogeneous mesh of F-actin as seen by electron microscopy (Fig. S2) and confocal microscopy (Fig. 3, A and B). For \( R_F > 0.01 \), large bundles appear within the mesh (Fig. 3, C and D). The value of \( R_F \approx 0.01 \) above which bundles appear is roughly independent of \( L \) (19). From electron microscopy, the bundles appear as loose, branching structures with diameters \( \sim100 \) nm (Fig. 3 C, inset). These observations are consistent with reports for networks with filamin from chicken gizzard \( (22,32,33) \). We confirm this bundling transition by tracking the thermal motion of particles within the networks (Fig. S3).

Varying \( L \) has little effect on the visual appearance of the nonbundled networks (Fig. 3, A and B). However, in the bundled networks, F-actin partitions more readily into the bundles at high \( R_G \), forming networks of pure bundles without a background F-actin mesh, as visible in confocal microscopy (Fig. 3 D) or detectable by particle tracking (data not shown). This may be due to increased diffusion and decreased entanglements for shorter filaments, allowing them to more easily associate into bundles \( (32) \).

Linear response

To probe the mechanical properties of the filamin-gelsolin-F-actin networks, we use a stress-controlled rheometer. For
an actin concentration $c_A = 0.5$ mg/mL and $L = 15 \mu$m, a weakly cross-linked network having $R_F = 0.001$ is a soft, viscoelastic solid (Fig. 4 A, squares). The elastic modulus $G'$ is two- to threefold larger than the viscous modulus $G''$, and $G'(t)$ increases as a weak power-law with the frequency, $\omega$, over a broad frequency range. This network is only slightly stiffer than purely entangled actin (triangles). Increasing $R_F$ further to 0.01 only modestly increases $G'$ and has little impact on the frequency response.

This is in contrast to F-actin with rigid cross-linking induced by addition of NeutrAvidin to networks with a small fraction, $R_B$, of biotinylated actin monomers incorporated into the F-actin. Increasing $R_B$ leads to a drastic increase in the stiffness of the network, as shown in Fig. 4 B; this is accompanied by a decrease in the slope of the weak power-law frequency response of $G'(t)$, consistent with more solid-like behavior (Fig. S4).

In the filamin networks, as we systematically decrease the mean filament length $L$ from 10 to 2 $\mu$m by adding increasing amounts of gelsolin, $G'$ decreases from 1 to 0.2 Pa (Fig. 4 C). For rigidly cross-linked networks, $G'$ also decreases with $L$ (Fig. 4 D). For both types of cross-links, the slope of $G'(t)$ does not vary drastically with $L$.

**Dependence of the linear modulus on filament length**

To quantify the changes in the elasticity of these networks as we decrease $L$, we plot $G_o$ defined as

$$G_o = G'|_{\omega = 0.1 \text{ Hz}},$$

as a function of $L$ (Fig. 5 A). For $R_F = 0.001$, $G_o$ is 0.2 Pa for the networks with the shortest filaments, $L = 1$–2 $\mu$m. As we increase $L$ to 15 $\mu$m, $G_o$ increases to 0.5 Pa. For increasing values of $R_F$, $G_o$ starts out at roughly the same value for short filaments, but increases more strongly with $L$. Interestingly, at each $R_F$, $G_o$ increases stronger than linearly with $L$.

This strong dependence on $L$ is not expected from an affine theory (11) that has been used to describe the linear and nonlinear elasticity of actin cross-linked with pointlike rigid cross-links such as heavy meromyosin and scrulin (5,7,12). In this theory, network elasticity is governed by the thermal compliance of the semiflexible F-actin polymers; thermal fluctuations of the F-actin get stretched-out as the network is deformed (11). This model predicts

$$G_o = 6\rho k_B T \frac{l_p^2}{l_c^3},$$

where $\rho$ is the linear density of polymer, $k_B$ is Boltzmann’s constant, $T$ is the temperature, $l_p$ is the persistence length of F-actin, and $l_c$ is the distance between cross-links. Thus,
this theory the network elasticity is controlled by the distance between cross-links rather than by the length of the actin filaments, in disagreement with our results for filamin-F-actin.

Alternatively, the elasticity of our networks can originate from the compliant nature of the filamin cross-links. The large 160-nm chain between the actin binding domains of a filamin protein is quite flexible and can be modeled as a linear polymer with \( l_p = 20 \, \text{nm} \) \((34)\). As a result, a filamin cross-link is soft compared to an F-actin segment of length \( l_c \), which ranges from 0.3 to 2 \( \mu \text{m} \). This suggests a model in which the compliance of the network is governed by the flexible cross-links. The actin polymers are treated as rigid rods linked by many flexible linkers, as depicted in Fig. 1 A. When the network surrounding a rigid rod is deformed, the linkers get stretched by an amount that increases linearly in the distance from the center of the rod, as shown in Fig. 1 C. Provided the network deformation is uniform on the length scale of \( L \),

\[
G_o = \frac{1}{8} \rho n k l \sim R_F L^2,
\]

with \( k \) the stiffness of the flexible cross-links and \( n \) the average number of cross-links per actin filament \((21)\). The explicit \( L \) dependence arises as a direct result of the nonuniform deformation profile of the cross-links. The average number of cross-links per actin filament is proportional to both \( R_F \) and \( L \) and is given by \( n = 370 R_F L \); thus, the overall prediction is that \( G_o \) will increase proportional to \( R_F L^2 \).

To test this mechanism, we plot \( G_o \) as a function of \( R_F L^2 \) (Fig. 5 C). The data for different cross-linking densities collapses onto a single curve. For \( R_F L^2 \geq 0.1 \), \( G_o \) scales nearly linearly with \( R_F L^2 \), consistent with the prediction. This supports the model of crosslink-dominated elasticity. Below \( R_F L^2 = 0.1 \), which corresponds to \( n = 7 \) cross-links for a 5-\( \mu \text{m} \) filament, the values of \( G_o \) are roughly equivalent to the elasticities we measure for F-actin solutions in the absence of cross-linking, as shown in Fig. 5 C (shaded bar). This suggests that the linear elasticity of these weakly cross-linked networks is dominated by the solution elasticity, not by the cross-links. The threshold of \( R_F L^2 = 0.1 \) corresponds to typical physiological conditions \((L = 2 \, \mu \text{m}, R_F = 0.02)\) \((35,36)\), suggesting that by spatially or temporally regulating \( L \), cytoskeletal elasticity could be adjusted from essentially that of entangled F-actin to a network with tunable stiffness.

By contrast, the dependence of \( G_o \) on \( L \) for the rigidly cross-linked networks is of a qualitatively different form; \( G_o \) increases linearly with \( L \) for small \( L \) but approaches a plateau for large \( L \) (Fig. 5 B). Simulations of two-dimensional \((37)\) and three-dimensional \((38)\) stiff polymer networks reveal a dependence on \( L \) qualitatively similar to our results. The departure from the plateau for decreasing \( L \) in simulations has been attributed to an increase in the nonaffinity in the deformation of the network, where the affine thermal theory is expected to break down.

### Nonlinear response

The dependence of \( G_o \) on \( R_F \) and \( L \) is consistent with network elasticity that is governed by the filamin cross-links. We further test the origin of the elasticity by measuring the nonlinear elastic properties of the filamin-F-actin gels with two complementary techniques—strain ramps and prestress measurements.

#### Strain ramps

In the first approach, we increase the strain, \( \gamma \), at a fixed rate and measure the resulting stress, \( \sigma \). From the derivative of the stress-strain curve, \( K = d\sigma/d\gamma \), we quantify the nonlinear behavior. This technique has been used to study nonlinear behavior of both entangled and cross-linked F-actin networks \((39–41)\). For a filamin cross-linked network with \( L = 15 \, \mu \text{m} \) and \( R_F = 0.01 \), \( K \) normalized by its initial value, \( K_o \), is equal to 1 for small strains (Fig. 6 A). At the critical strain, \( \gamma_c = 0.06 \), \( K/K_o \) increases above 1, and the network begins to stiffen. It stiffens 30-fold before breaking at \( \gamma_m = 0.9 \). Networks with shorter filaments initially display weakening behavior, where \( K/K_o \) decreases below 1, due to their lower network connectivity, but eventually stiffen. As we increase \( L \), \( \gamma_c \) increases markedly, as shown in Fig. 6 A.

By contrast, rigidly cross-linked networks with \( L > 5 \, \mu \text{m} \) stiffen at small strains, independent of \( L \) (Fig. 6 B). Networks with \( L \leq 2 \, \mu \text{m} \) do not stiffen and display weakening behavior. This is consistent with a transition from stiffening behavior arising from pulling out fluctuations in F-actin, where
The 1/L-dependence arises because the amount an end-bound cross-link must stretch to accommodate a given macroscopic network strain increases with the length of the rigid rod to which it is bound.

Plotting $\gamma_c$ as a function of $L^{-1}$ in Fig. 7 A, the dependence of $\gamma_c$ on $L^{-1}$ is in stark contrast to the stiffening behavior of rigidly cross-linked networks, which display no dependence of $\gamma_c$ on $L$. The increase of $\gamma_c$ with increasing $L^{-1}$ is qualitatively consistent with the prediction of the model. We see similar behavior for $\gamma_m$ (Fig. 7 B), suggesting that the nonuniform deformation profile of the linkers prevails up to large strains. Interestingly, the $\gamma_c$ data from the two systems converge at small $L^{-1}$ (Fig. 7 A). In this limit of large $L$, the model of rigid rods with flexible linkers predicts that the smallest of strains would lead to stiffening. However, this model relies on the linkers being the softest mode in the system. When the prediction for stiffening by the linkers would yield a lower $\gamma_c$ than by the F-actin segments themselves, this picture breaks down, and it is no longer valid to assume the F-actin behave as rigid rods. In this limit, the compliance of the F-actin would

\[ \gamma_c = \frac{1}{6} \left( \frac{l_c}{l_p} \right) \]

is set only by $l_c$ and $l_p$, to weakening behavior, where the network becomes too sparsely connected to stiffen. However, the strong dependence of $\gamma_c$ on $L$ for filamin-F-actin is inconsistent with such a nonlinear response arising from thermal fluctuations of the actin filaments being stretched-out.

We propose instead that the nonlinear response for filamin-F-actin originates from the stiffening behavior of the cross-links. Single molecule experiments indicate that filamin proteins stiffen markedly as they are stretched toward their contour length $l_o$ (42). When the network surrounding a rigid rod is deformed strongly, linkers bound at the ends of the polymers are stretched most, as depicted in Fig. 1 C. These linkers will be the first to reach full extension and stiffen, setting the critical strain at which the network begins to stiffen. These end-bound linkers reach full extension at a strain (21):

\[ \gamma_c = 4 \frac{l_o}{L}. \]
contribute to the stiffening behavior of the system, consistent with our observation.

Prestress measurements

In our second technique for probing nonlinear response, we apply a steady prestress, $\sigma_0$, and probe the differential elastic modulus, $K'(\sigma_0, \omega)$, with a small oscillatory stress. This technique has been used in cross-linked F-actin networks to study nonlinear stiffening behavior (5,6,12,41). Rigidly cross-linked F-actin networks display stiffening with $K' \sim \sigma^{3/2}$ (5); we see the same behavior for networks cross-linked by biotin-NeutrAvidin (Fig. 8 B). This is consistent with the predictions for the affine thermal model in which the nonlinear response is due to pulling out thermal bending fluctuations in the semiflexible actin filaments within the network (5,11).

Our model of rigid filaments connected by multiple flexible linkers predicts a qualitatively different stiffening behavior that arises from the stiffening of the filamin cross-links. The theoretical model is extended to the nonlinear regime by employing a self-consistent effective medium approach (19–21). The linkers are bound on one side to the rigid rod and on the other to an elastic continuum with a nonlinear elasticity that is required to self-consistently represent a uniform and isotropic collection of such elements. As the network is deformed, the linkers get stretched and stiffen one-by-one as they approach full extension and start pulling back on the effective medium. This model predicts $K' \sim \sigma_0$ in the limit of a dense network.

To test this prediction, we measure $K'(\sigma_0)$ for the networks cross-linked by filamin. For a network with $R_F = 0.003$ and $L = 15 \mu$m (solid circles), $K'$ increases with $\sigma_0$ above a critical stress, $\sigma_c = 0.1$ Pa, and reaches a stiffness, $K_m = 10$ Pa, before breaking at $\sigma_m = 1$ Pa. Networks with higher $R_F$ and $L$ stiffen more and support larger stresses. For these networks, $K'$ increases more strongly than linear in $\sigma_0$ just above $\sigma_c$, whereas at high $\sigma_0$, $K' \sim \sigma_0$ (Fig. 8 A). This unusual stiffening behavior is in agreement with the prediction of the model. Rescaling $K'$ by its initial value and $\sigma_0$ by $\sigma_c$, the $K'(\sigma_0)$ data for networks formed with different $R_F$ and $L$ collapse onto a single curve, provided the network is not highly bundled (Fig. 9). Our rescaled data agrees well with the nonlinear response calculated with the effective medium model (Fig. 9), with only one fit parameter that represents the coupling of a rigid rod to the effective medium. In contrast, the rescaled data from networks rigidly cross-linked by biotin-NeutrAvidin fall on a separate curve, which is well described by the prediction of the affine thermal theory of cross-linked semiflexible networks (Fig. 9). These data support the model of crosslink-dominated elasticity in the filamin-F-actin networks.

Interestingly, although the filamin-F-actin networks are all quite compliant, the maximum stiffness before breaking, $K'_m$, increases strongly with $R_F$, presumably because network failure is due to filamin unbinding (19,43). Thus,
the overall magnitude of stiffening, $K_m'/G_o$, increases with $R_F$ (Fig. 8 A and Fig. S6 B). Of these networks the highly bundled ones show the most dramatic stiffening (open symbols). The opposite behavior is observed for the rigidly cross-linked networks; $G_o$ increases significantly with $R_B$, while $K_m'$ is nearly independent of $R_B$ (Fig. 8 B and Fig. S6 B), presumably because network failure is due to F-actin rupture (12).

Dependence of the maximum stress on filament length

Assuming cross-link unbinding as the dominant failure mode for these networks, a scaling argument based on the theoretical model predicts how $\sigma_m$ scales with $c_A$, $R_F$, and $L$. Essentially, each cross-link will unbind from F-actin at a force, $f_m$. With multiple cross-links per filament, the cross-links act in parallel, and the total rupture force per filament increases linearly with $n$. From the density of filaments and assuming an isotropic orientation of filaments within the network, the maximum stress is (20,21)

$$\sigma_m = \frac{1}{45} \rho n f_m.$$  

Thus, as we increase $L$ at fixed $R_F$, the number of filamins per actin filament will increase, and the prediction is

$$\sigma_m \sim n \sim R_F L.$$  

We first look at $\sigma_m$ measured in prestress experiments. In Fig. 8 A, we see that $\sigma_m$ supported by the $R_F = 0.003$ networks increases as we increase $L$. To quantify this, we plot $\sigma_m$ as a function of $L$ in Fig. 10 A. Above a critical value of $L$, the maximum stress increases with $L$. This critical value of $L$ decreases with increasing $R_F$. Similarly, for fixed $L$, $\sigma_m$ increases roughly linearly with $R_F$ over a broad range of $R_F$ (Fig. S6 A). For the highest values of $R_F$, where the networks are highly bundled, $\sigma_m$ increases dramatically (open symbols, Fig. S6 A). For $R_F$ below a critical value, $\sigma_m$ is roughly independent of $R_F$. This critical value of $R_F$ decreases with increasing $L$ (19).

We can collapse all the filamin data onto a single curve by plotting $\sigma_m$ as a function of $R_F L$ (19), as shown in Fig. 10 B. For $R_F L > 0.01$, $\sigma_m$ grows nearly linearly with $R_F L$, consistent with the prediction of the model. For smaller values of $R_F L$, the network is rather weakly connected and breaks at very low levels of stress. The value of $R_F L \approx 0.01$ corresponds to $n \approx 4$. At physiological conditions, $n \approx 15$—suggesting that the cytoskeleton operates in a regime where it has high enough connectivity to support large external stresses or internal tensions compared to purely entangled F-actin without rupturing. The linear scaling with $R_F L$ or $n$ suggests that failure of these networks is indeed determined by unbinding of cross-links (19,43). In contrast, $\sigma_m$ for rigidly cross-linked networks is nearly independent of $R_B$ for $R_B > 0.001$ (Fig. S6 A); this is consistent with rupture of F-actin at network failure (12).

We can also determine $\sigma_m$ from the strain ramp measurements. In Fig. 6 A (inset), we see that $\sigma_m$ also increases with $L$ for strain ramps conducted at a rate of 0.1/s. Plotting $\sigma_m$ determined in this way as a function of $L$ (squares, Fig. 10 A) for $R_F = 0.01$, we find that these measurements of $\sigma_m$ show similar scaling to the prestress measurements, and at this strain rate the values from the two methods nearly match. More generally, we expect that $\sigma_m$ will depend on the rate of the measurement. The unbinding force for a single cross-link is expected to increase as the logarithm of the loading rate (40,44). In an analogous macroscopic measurement, we find that $\sigma_m$ increases as the logarithm of the loading rate on the network, again consistent with cross-link unbinding at network failure (inset, Fig. 10 B).

CONCLUSIONS

The linear and nonlinear elastic behavior of filamin-gelsolin-F-actin networks support a model of crosslink-dominated...
elasticity. The F-actin behaves as a rigid filament that constrains the deformation profile of the flexible cross-links bound along its length; this leads to the unusual $L$ dependence in the rheology of these networks. Our data suggest that the lengths of actin filaments within cross-linked cytoskeletal networks may be an important determinant of cell mechanics.

Large, flexible cross-links like filamin form compliant gels that can nonetheless support stresses that are orders-of-magnitude larger than those of purely entangled F-actin. The stiffness of these networks can be tuned over a broad range by external stress or internal tension (3,6,19). In contrast, rigid cross-links form networks with a linear stiffness that is highly tunable by increasing the cross-link concentration, but show less dramatic nonlinear stiffening and tend to break at smaller strains (5,7). Interestingly, the mechanical response of F-actin networks can be tuned between these two cases by systematically varying the molecular weight of a cross-link (8).

Many physiological cross-links are smaller and expected to be more incompliant than filamin; within the cell these cross-links typically organize F-actin into bundles rather than orthogonal meshworks. For example, the $\alpha$-actinin dimer forms an antiparallel rod of ≈30 nm, and fimbrin has two actin binding domains in tandem and is only ≈12 nm. Indeed, rheological studies show that $\alpha$-actinin-F-actin networks have highly tunable linear stiffness (45), suggesting $\alpha$-actinin behaves predominantly as a rigid cross-link. This also suggests that the cell may use large, compliant cross-linking proteins like the 160-nm-long fimbrin dimers precisely because of the unique mechanical properties of the networks they form. In support of this view, filamin-F-actin networks mimic many key rheological features of cells (6,18). This highlights the potential value of these results in providing insight into the behavior observed in cells.

SUPPORTING MATERIAL

Six figures are available at http://www.biophysj.org/biophysj supplemental/S0006-3495(10)00737-X.

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REFERENCES


